Chapter 18: Hints and Selected Solutions

Section 18.1 (page 498)

18.2

1. This is an allowable change. The language in question does not have any position predicates, so the model for the original world and the modified world are identical.
4. If we make the large cube a dodecahedron, then b_3 would no longer be in the extension of cube, so the model would be different. Hence this is not an allowable change.
7. If we add a dodecahedron to the world, the domain will have to have five objects in it, not just four, so the model would change. Hence this is not an allowable change.

18.3 Hint: The extension *Bet* of Between would be $\langle b, b2, b3 \rangle$.

18.4 Hint: With two objects in the domain, there are four ordered pairs, and so $2^4 = 16$ sets of ordered pairs. Your task it to list them all.

18.5

1. *Every*
4. *Exactly one*

18.6

1. Conservativity
4. Monotone decreasing

Section 18.2 (page 507)

18.7

1. $g[\mathsf{y}/c]$ is the function which assigns b to x and c to y.
4. $g[\mathsf{x}/b]$ is the same as the original function g. That is, it assigns b to x but that is all.

18.8

1. Appropriate assignments: those that assign values to variables including both y and z.
   Satisfying assignments: those that assign 1 to y and 2 to z, and those that assign 2 to y and 3 to z.

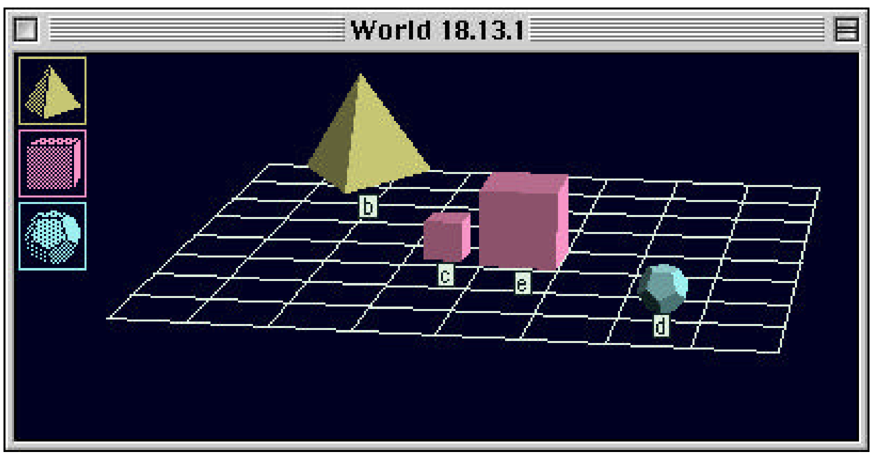
4. Appropriate assignments: Those that assign values to variables including x.
   Satisfying assignments: there are none.

7. Appropriate assignments: those that assign values to variables including x, y, and z.
   Satisfying assignments: those appropriate assignments that assign 1 to y and 2 to z, and those that assign 2 to y and 3 to z.

10. Appropriate assignments: since there are no free variables, all variable assignments are appropriate for this wff.
    Satisfying assignments: since the wff is false in the structure, there are no variable assignments that satisfy the wff.

18.11 Hint: use induction on wffs, except for the base case, use any sentence that is atomic or starts with a quantifier.

18.13



Section 18.3 (page 512)

18.14 The inductive step for $\wedge$ **Intro** is very easily obtained by modifying the one for $\rightarrow$ **Elim** as follows:

> $\wedge$ **Intro**: Suppose the n^{th} step derives the sentence $Q \wedge R$ from an application of $\wedge$ **Intro** to sentences Q and R appearing earlier in the proof. Let $A_1, \ldots, A_k$ be a list of all the assumptions in force at step n. By our induction hypothesis we know that Q and R are both established at valid steps, that is, they are first-order consequences of the assumptions in force at those

steps. Furthermore, since $\mathcal{F}$ only allows us to cite sentences in the main proof or in subproofs whose assumptions are still in force, we know that the assumptions in force at steps Q and R are also in force at $\mathsf{Q} \wedge \mathsf{R}$. Hence, the assumptions for these steps are among $\mathsf{A}_1, \ldots, \mathsf{A}_k$. Thus, both Q and R are first-order consequences of $\mathsf{A}_1, \ldots, \mathsf{A}_k$. We now show that $\mathsf{Q} \wedge \mathsf{R}$ is a first-order consequence of $\mathsf{A}_1, \ldots, \mathsf{A}_k$.

Suppose $\mathfrak{M}$ is a first-order structure in which all of $\mathsf{A}_1, \ldots, \mathsf{A}_k$ are true. Then we know that $\mathfrak{M} \models \mathsf{Q}$ and $\mathfrak{M} \models \mathsf{R}$, since these sentences are first-order consequences of $\mathsf{A}_1, \ldots, \mathsf{A}_k$. But in that case, by the definition of truth in a structure we see that $\mathfrak{M} \models \mathsf{Q} \wedge \mathsf{R}$ as well. So $\mathsf{Q} \wedge \mathsf{R}$ is a first-order consequence of $\mathsf{A}_1, \ldots, \mathsf{A}_k$. Hence, step n is a valid step.

Section 18.4 (page 514)

18.18 Hint: There is exactly one way to interpret SameShape so as to make all ten axioms true. What is it? For example, 2 and 5 are both in the extension of Tet so they will be the "same shape". By contrast, 4 is not in the extension of Tet so it will not be the "same shape" as 2 and 5. You need to figure out just what makes two numbers be in the extension of the same shape predicates and so count as being of the "same shape."

Section 18.5 (page 516)

18.21

1. If we use $f(z) = z^2$ the sentence would be true iff $1 + z^2 < z^2$, for all natural numbers z. But in fact it is false for all such z so we cannot use this as a Skolem function for this sentence.

4. If we use $f(z) = z^3$ the sentence would be true iff $1 + z^2 < z^3$, for all natural numbers z. This is true of all natural numbers except for 0 and 1. But even one counterexample shows that we cannot use this function as a Skolem function for this sentence.

Section 18.6 (page 519)

18.22

1. The name Max is not of the right form to be unifiable with father(x), since there is no substitution for the variable x that makes the two terms the same. Note that it has nothing to do with the fact that Max himself is not a father.

4. The terms father(x) and father(mother(claire)) are unifiable, by means of the substitution that substitutes mother(claire) for the variable x.

18.24 You are asked for a set of four terms. Here is a set with two:

$$\{\mathsf{h}(\mathsf{y}, \mathsf{z}), \mathsf{h}(\mathsf{f}(\mathsf{x}), \mathsf{g}(\mathsf{a}))\}$$

Section 18.7 (page 524)

18.27 Hint: Either Quaid admires himself or he doesn't. Break into two cases depending on which of these is the case.