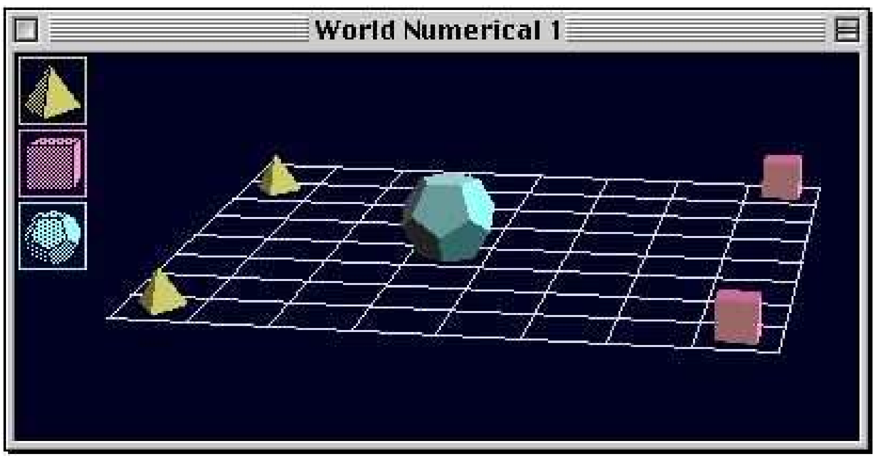
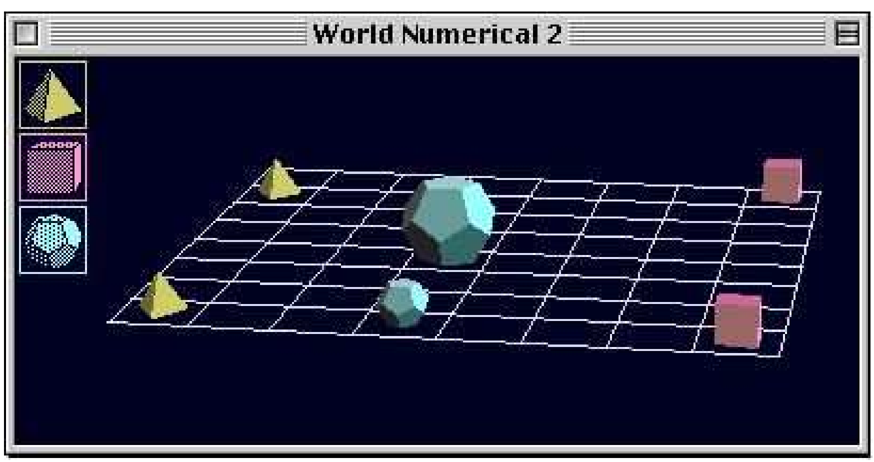
Chapter 14: Hints and Selected Solutions

Section 14.1 (page 371)

14.1 One way to do the **You Try It** is shown in the two worlds below. You should submit different looking worlds.





14.2 1. *There is exactly one tove.*

4. *All toves are identical (=).*

14.3 1. $\exists x \exists y\ (\mathsf{Dodec}(x) \land \mathsf{Dodec}(y) \land x \neq y)$

4. $\exists x \exists y \exists z\ [x \neq y \land x \neq z \land y \neq z \land \neg\mathsf{Small}(x) \land \neg\mathsf{Small}(y) \land \neg\mathsf{Small}(v) \land \forall v(\neg\mathsf{Small}(v) \rightarrow (v = x \lor v = y \lor v = z))]$

14.6 1. $\exists x \exists y\ (\mathsf{Tet}(x) \land \mathsf{Tet}(y) \land x \neq y)$

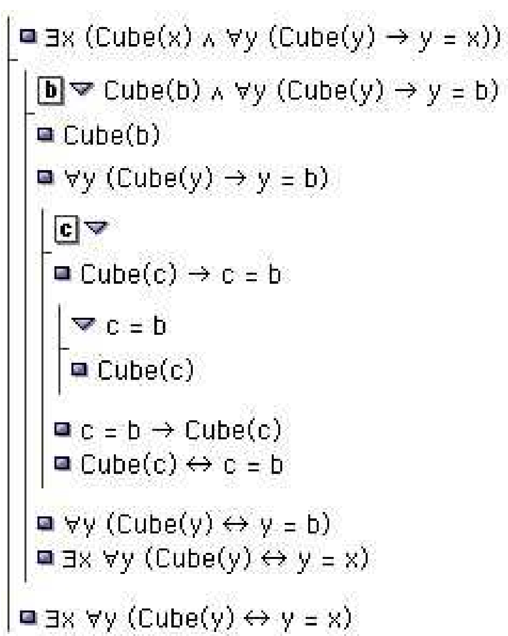
4. $\exists x \exists y \exists z\ [x \neq y \land x \neq z \land y \neq z \land \mathsf{Dodec}(x) \land \mathsf{Dodec}(y) \land \mathsf{Dodec}(z)]$

7. $\forall x\ (\mathsf{Tet}(x) \rightarrow \exists y \mathsf{BackOf}(x, y))$

10. $\forall x\ (\mathsf{Dodec}(x) \rightarrow (\mathsf{Small}(x) \lor \mathsf{Medium}(x) \lor \mathsf{Large}(x))$

Section 14.2 (page 377)

14.10 Hint (fill in the justifications if you use this proof):



14.14 Hint: The argument is not valid.

14.15 Hint: This argument *is* valid.

14.17 Correction: This problem has an extra negation in its statement. It should be stated as follows:

$\neg\exists^{\geq n+1} x\, S(x) \leftrightarrow \exists^{\leq n} x\, S(x)$

14.22 We want to give an informal proof of $\exists! x\, [x^2 - 2x + 1 = 0]$.

Existence: We first prove that there is an x that satisfies $x^2 - 2x + 1 = 0$. To prove this, we note that $1^2 - 2 \times 1 + 1 = 0$ so 1 satisfies the wff.

Uniqueness: We now suppose that x is any number that satisfies $x^2 - 2x + 1 = 0$. Since $x^2 - 2x + 1 = (x-1)^2$ we see that $(x-1)^2 = 0$. But then $x - 1 = 0$ so $x = 1$. Hence we see that 1 is the only number that satisfies the equation.

Section 14.3 (page 382)

14.28

1. $\exists x \exists y\, [\mathsf{Cube}(x) \land \mathsf{Cube}(y) \land x \neq y \land \forall z\, (\mathsf{Cube}(z) \rightarrow (z = x \lor z = y)) \land \mathsf{Medium}(x) \land \mathsf{Medium}(y)]$
4. $\exists x \exists y\, [\mathsf{Cube}(x) \land \mathsf{Cube}(y) \land x \neq y \land \forall z\, (\mathsf{Cube}(z) \rightarrow (z = x \lor z = y)) \land$
   $\exists u \exists v\, [\mathsf{Dodec}(u) \land \mathsf{Dodec}(v) \land u \neq v \land \forall z\, (\mathsf{Dodec}(z) \rightarrow (z = u \lor z = v)) \land$
   $\mathsf{LeftOf}(x, u) \land \mathsf{LeftOf}(x, v) \land \mathsf{LeftOf}(y, u) \land \mathsf{LeftOf}(y, v)]]$

Section 14.4 (page 388)

14.31

1. Few x [Cube(x), Small(x)]
   This sentence is false in Cooper's World since well over 10% of the cubes are small.
4. Few x SameCol(x, b)
   This sentence is false in Cooper's World since 1/6 of the blocks are in the same column with *b*, and that is more than 10%. This is a bit tricky, since you might well forget to count *b* itself. If the sentence said that few blocks other than *b* are in the same column as *b*, it would be true.
7. ¬∃x Most y Adjacent(x, y)
   This sentence is true in Cooper's world since nothing is adjacent to more than 1/6 of the blocks, well less than half.

14.32

1. *Most tetrahedra are small.* (True)
4. *Most tetrahedra are adjacent to something.* (True)
7. *Some block is adjacent to most cubes.* (False)
10. *Most cubes are in front of most tetrahedra.* (True)

Section 14.5 (page 395)

14.33 This argument is valid, by the conservativity of the determiner *few*.

14.37 Since anything that is large is not small, this argument is valid by the monotone decreasing property of *few*.

14.41 Let's give an informal proof of the argument:

$$\exists x\,[\mathsf{Dodec}(x) \land \mathsf{Most}\,y\,(\mathsf{Dodec}(y), y = x))]$$

$$\exists! x\,\mathsf{Dodec}(x)$$

> **Proof:** By the premise, there is a dodecahedron that is identical to most dodecahedra. Let's call one of these *b*. But if there were even one more dodecahedron, then at most half of the dodecahedra would be *b*. So the *b* must be the only dodecahedron. Hence there is a unique dodecahedron.

14.45 The conclusion of this argument is a logical truth, so the argument is valid simply because the conclusion can never be false.

Section 14.6 (page 398)

14.57

1. $\exists x \exists y\,[\mathsf{Pet}(x) \land \mathsf{Pet}(y) \land x \neq y \land \mathsf{Gave}(\mathsf{max}, x, \mathsf{claire}, 2:00) \land \mathsf{Gave}(\mathsf{max}, y, \mathsf{claire}, 2:00)]$

4. This cannot be translated into our language, for the simple reason that we treated $\mathsf{Student}(x)$ as a unary predicate. If we had treated it as a binary predicate $\mathsf{Student}(x, t)$, then we could have translated it.

7. Hint: This can be translated.

10. We could translate the closely related sentence *If someone gave Max a pet, it must have been Claire*, but the *must* in 10 brings in an element of necessity that cannot be expressed in our language or any simple extension of it.

15. Hint: Here the problem is the *should*.