Chapter 11: Hints and Selected Solutions

Section 11.1 (page 291)

11.4 1. $\forall x \forall y\ [(\mathsf{Small}(x) \wedge \mathsf{Large}(y) \rightarrow \mathsf{FrontOf}(x, y))$

4. $\neg \forall x \forall y\ ((\mathsf{Tet}(x) \wedge \mathsf{Tet}(y)) \rightarrow \mathsf{SameCol}(x, y))$

7. $\exists x \exists y\ (\mathsf{Tet}(x) \wedge \mathsf{Tet}(y) \wedge x \neq y \wedge \mathsf{SameSize}(x, y))$

11.7 2. *Some of the parties are not lonely.*

4. *There is a lonely party* or *Someone went to a lonely party.*

7. *There is only one party* or*Everyone went to the same party*

9. *No one was at a party with everyone else.*

Section 11.2 (page 296)

11.10 2. The person intended to say that every two cubes has something between them, but instead said that every cube has something between it and every cube. This would imply that a cube has something between itself and itself, which is false. So you need to fix the sentence up to say what was intended.

5. This sentence says that everything that has something between it and c is not large. But this is false because of the large cubes in the back row. They have a block between them and c. To make this sentence true in an interesting way, we might say that every block that does *not* have a block between it and c is not large.

8. The sentence says that the only block with nothing to its left is a. This is false since there are in fact three blocks that have nothing to their left. There is no way to make this sentence true in a nonvacuous way by adding negations inside. You can make it true in a vacuous way by adding a negation sign in front of $\mathsf{LeftOf}(y, x)$. This sentence then says that anything with everything to its left is a. This is vacuously true because there is nothing that has everything to its left.

11. This sentence says that the only thing that is between two cubes is b. This is clearly false. Again, there is no way to make it non-vacuously true by adding negations to atomic sentences. But you can make it true by adding a negation in front of, say, $\mathsf{Cube}(\mathsf{x})$. Do you understand why it is true?

14. This says that all the small dodecahedraare identical. It is true simply because there are no small dodecahedra.

17. This sentences claims that b is the only small tetrahedron, which is true.

11.13 Here are some hints. Before starting to build your world, try to understand what the sentences say. You might even want to translate them into English. Then, start to draw out some consequences of the sentences. For example, it is pretty easy to see that b must be a medium dodecahedron, while c must be a large dodecahedron. Write down as many consequence of the sentences as you can think of before starting to build your world. These will guide you in building the world.

Section 11.3 (page 299)

11.17 1. $\forall \mathsf{x} \forall \mathsf{y}\ ((\mathsf{Tet}(\mathsf{x}) \wedge \mathsf{Dodec}(\mathsf{y})) \rightarrow \mathsf{FrontOf}(\mathsf{x}, \mathsf{y}))$

4. $\forall \mathsf{x}(\mathsf{Dodec}(\mathsf{x}) \rightarrow \exists \mathsf{y}(\mathsf{Cube}(\mathsf{y}) \wedge \mathsf{SameSize}(\mathsf{x}, \mathsf{y})))$

7. $\forall \mathsf{x}((\mathsf{Cube}(\mathsf{x}) \wedge \exists \mathsf{y}\mathsf{BackOf}(\mathsf{y}, \mathsf{x})) \rightarrow \mathsf{Small}(\mathsf{x}))$

10. $\forall \mathsf{x}((\mathsf{Dodec}(\mathsf{x}) \wedge \exists \mathsf{y}(\mathsf{Cube}(\mathsf{y}) \wedge \mathsf{LeftOf}(\mathsf{x}, \mathsf{y}))) \rightarrow \mathsf{Large}(\mathsf{x}))$

Section 11.4 (page 302)

11.18 1. Paraphrase: *Every object that has nothing in front of it is large.*
Translation: $\forall \mathsf{x}\ (\neg \exists \mathsf{y}\mathsf{FrontOf}(\mathsf{y}, \mathsf{x}) \rightarrow \mathsf{Large}(\mathsf{x}))$

4. Paraphrase: *For any two objects, if* ***e*** *is between them, then they are both small.*
Translation: $\forall \mathsf{x} \forall \mathsf{y}\ (\mathsf{Between}(\mathsf{e}, \mathsf{x}, \mathsf{y}) \rightarrow (\mathsf{Small}(\mathsf{x}) \wedge \mathsf{Small}(\mathsf{y})))$, or, if you prefer to make the fact that they two objects have to be distinct explicit, $\forall \mathsf{x} \forall \mathsf{y}\ ((\mathsf{Between}(\mathsf{e}, \mathsf{x}, \mathsf{y}) \wedge \mathsf{x} \neq \mathsf{y}) \rightarrow (\mathsf{Small}(\mathsf{x}) \wedge \mathsf{Small}(\mathsf{y})))$

11.20 1. $\forall \mathsf{x}\ (\mathsf{LeftOf}(\mathsf{x}, \mathsf{a}) \rightarrow \neg \forall \mathsf{y}\ (\mathsf{LeftOf}(\mathsf{y}, \mathsf{b}) \rightarrow \mathsf{Larger}(\mathsf{x}, \mathsf{y})))$

4. $\forall \mathsf{x}\ (\mathsf{LeftOf}(\mathsf{x}, \mathsf{a}) \rightarrow \exists \mathsf{y}\ (\mathsf{Smaller}(\mathsf{x}, \mathsf{y}) \wedge \forall \mathsf{z}\ ((\mathsf{Cube}(\mathsf{z}) \wedge \mathsf{RightOf}(\mathsf{z}, \mathsf{b})) \rightarrow \mathsf{BackOf}(\mathsf{y}, \mathsf{z})))$

7. This sentence on its own seems ambiguous. It might be used to mean that only dodecahedra are larger than everything of a different shape. It might also be used to mean that only dodecahedra are larger than every other block. This is equivalent to saying that

if there is a largest block, it must be a dodecahedron. Both of these interpretations are true in Gödel's World, but somehow the second seems like the most likely thing to have been meant here. That is the one whose translation we give here:
$\forall x\ (\forall y\ (y \neq x \rightarrow \mathsf{Larger}(x, y)) \rightarrow \mathsf{Dodec}(x))$

10. $\forall x\ (\exists y \exists z\ \mathsf{Between}(x, y, z) \rightarrow \mathsf{Cube}(x))$

11.21

1. $\forall x\ (\mathsf{Student}(x) \rightarrow \exists y \exists z \exists t\ (\mathsf{Student}(y) \wedge x \neq y \wedge \mathsf{Pet}(z) \wedge \mathsf{Gave}(x, z, y, t)))$

4. $\neg \exists x (\mathsf{Student}(x) \wedge \forall y\ (\mathsf{Pet}(y) \rightarrow \exists t\ \mathsf{Fed}(x, y, t)))$

7. $\forall x \forall t\ (\mathsf{Pet}(x) \wedge \mathsf{Gave}(\mathsf{max}, x, \mathsf{claire}, t) \rightarrow (\mathsf{Owned}(\mathsf{claire}, x, t) \wedge \neg \mathsf{Owned}(\mathsf{max}, x, t))$

10. You are asked to translate the following pair of sentences: *Max gave Claire a pet between 2:00 and 3:00. It was hungry.* Notice that the *it* in the second sentence refers back to the pet referred to in the first sentence. To translate these into FOL, we will need to write just one sentence that expresses them both. There is a suggestion in the sentence that Max gave Claire a single pet, and that that pet was hungry. However, this is only an implicature, since we could add to the sentences another sentence saying *Max gave Claire another pet that was not hungry.* Also, there is an issue as to just when the pet Max gave Claire was hungry. The sentence seems to be saying that it was hungry when Max gave it to Claire. Thus, the right translation for the sentences is the following:
$\exists x\ (\mathsf{Pet}(x) \wedge \exists t (2:00 < t \wedge t < 3:00 \wedge \mathsf{Gave}(\mathsf{max}, x, \mathsf{claire}, t) \wedge \mathsf{Hungry}(x, t)))$

Section 11.8 (page 316)

11.40

1. $\neg \forall x\ (\mathsf{Cube}(x) \rightarrow \forall y\ (\mathsf{Tet}(y) \rightarrow \mathsf{Smaller}(x, y)))$

4. $\forall x\ ((\mathsf{Cube}(x) \wedge \forall y \neg \mathsf{BackOf}(y, x)) \rightarrow \neg \exists z (\mathsf{Cube}(z) \wedge x \neq z \wedge \mathsf{Smaller}(x, z))$

7. Hint: Another way of saying the same thing is that if one thing is in back of another, then the former is larger than the latter.

10. $\neg \exists x \exists y \exists z (y \neq z \wedge x \neq y \wedge x \neq z \wedge \mathsf{SameSize}(x, y) \wedge \mathsf{SameSize}(x, z))$