

## Chapter 10: Hints and Selected Solutions

### Section 10.1 (page 264)

**10.1** The following fills in some of the rows for you. Be sure you understand these.

	Annotated sentence	Truth-functional form	a/b/c
1.	$\overline{\forall x (x = x)}_A$	A	b
4.	$\overline{\forall x (\text{Cube}(x) \wedge \text{Small}(x))}_A \rightarrow \overline{\forall x (\text{Small}(x) \wedge \text{Cube}(x))}_B$	$A \rightarrow B$	b
7.	$\overline{[\forall z (\text{Cube}(z) \rightarrow \text{Large}(z))}_A \wedge \overline{\text{Cube}(b)}_B] \rightarrow \overline{\text{Large}(b)}_C$	$[A \wedge B] \rightarrow C$	b
10	$\overline{[(\forall u \text{Cube}(u))}_A \rightarrow \overline{\forall u \text{Small}(u)}_B] \wedge \overline{\neg \forall u \text{Small}(u)}_B] \rightarrow \overline{\neg \forall u \text{Cube}(u)}_A$	$[(A \rightarrow B) \wedge \neg B] \rightarrow \neg A$	a

**10.2** The argument is logically valid, but its truth functional form is as shown below, so it is not tautologically valid. (We have not shown the annotation, but you should be able to figure it out from the form.)

A $\wedge$ B
C $\wedge$ D
E $\wedge$ F

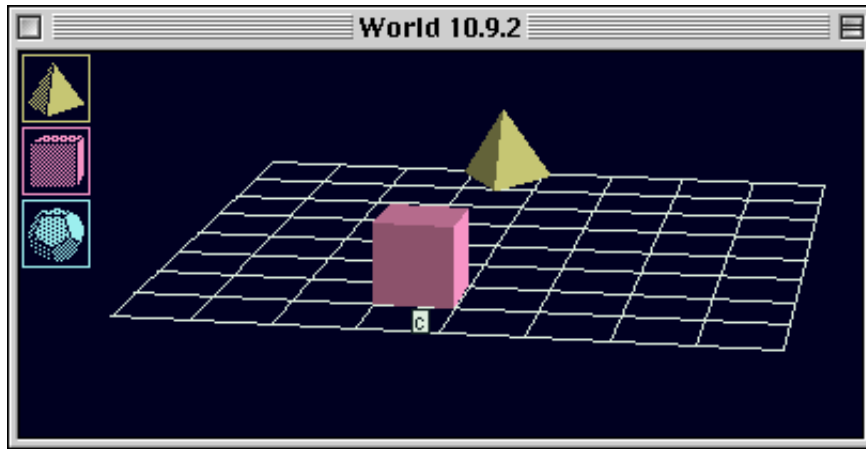
**10.4** The argument is tautologically valid. Here is its truth functional form:

A $\rightarrow$ B
$\neg$ B
$\neg$ A

Section 10.2 (page 273)

- |      |   |  |
|------|---|--|
| 10.8 | <ol style="list-style-type: none"> <li>1. <math>\exists x \text{ Dodec}(x) \rightarrow \exists y \text{ Small}(y)</math></li> <li>2. <math>\neg \exists y \text{ Small}(y)</math></li> <li>3. <math>\forall x (\text{Cube}(x) \rightarrow \text{Large}(x))</math></li> <li>4. <math>\text{Medium}(c)</math></li> </ol>  |  |
|      | <ol style="list-style-type: none"> <li>5. <math>\forall x \text{ Cube}(x) \rightarrow \text{Cube}(b)</math></li> <li>6. <math>\neg \exists y \text{ Dodec}(y)</math></li> <li>7. <math>\neg \exists x \text{ Dodec}(x)</math></li> <li>8. <math>\forall x (\neg \text{Large}(x) \rightarrow \neg \text{Cube}(x))</math></li> <li>9. <math>\neg \exists y \text{ Small}(y) \rightarrow \neg \exists x \text{ Dodec}(x)</math></li> <li>10. <math>\neg \text{Cube}(c)</math></li> </ol> | <p>✓ ▾ FO Con:</p> <p>✓ ▾ FO Con: 1,2</p> <p>✓ ▾ Taut Con: 2,1</p> <p>✓ ▾ FO Con: 3</p> <p>✓ ▾ Taut Con: 1</p> <p>✓ ▾ Ana Con: 3,4</p> |

10.9 The second sentence is not a logical truth. It is false in many worlds. One such is shown in below. You should submit a different world.



10.11 The following argument is of type (c), that is, it is valid but it is not FO valid:

$$\left| \begin{array}{l} \text{Cube}(a) \wedge \text{Cube}(b) \\ \text{Small}(a) \wedge \text{Large}(b) \end{array} \right. \left. \vphantom{\begin{array}{l} \text{Cube}(a) \wedge \text{Cube}(b) \\ \text{Small}(a) \wedge \text{Large}(b) \end{array}} \right| \exists x (\text{Cube}(x) \wedge \text{Large}(x) \wedge \neg \text{Smaller}(x, x))$$

Its truth functional form is

$$\begin{array}{|l} A \wedge B \\ C \wedge D \\ \hline E \end{array}$$

Here is a version with nonsense predicates:

$$\begin{array}{|l} P(a) \wedge P(b) \\ S(a) \wedge T(b) \\ \hline \exists x (P(x) \wedge T(x) \wedge \neg R(x, x)) \end{array}$$

To get a counterexample, let  $a = 1$ ,  $b = 2$ ,  $P(x)$  mean that  $x$  is a number,  $S(x)$  mean that  $x$  is odd, and  $T(x)$  mean that  $x$  is even, and let  $R(x, y)$  mean that  $x = y$ . Then both premises are true but the conclusion is false.

**10.15** The following argument is of type (b), that is, it is FO valid but not tautologically valid:

$$\begin{array}{|l} \text{Cube}(a) \\ \neg \text{Cube}(b) \\ \hline a \neq b \end{array}$$

Here is its truth functional form:

$$\begin{array}{|l} A \\ \neg B \\ \hline C \end{array}$$

Here is a version with nonsense predicates substituted:

$$\begin{array}{|l} P(a) \\ \neg P(b) \\ \hline a \neq b \end{array}$$

Remember here that we substitute for all predicates other than  $=$ .

**10.16** The following argument is tautologically valid; that is, it is of type (a).

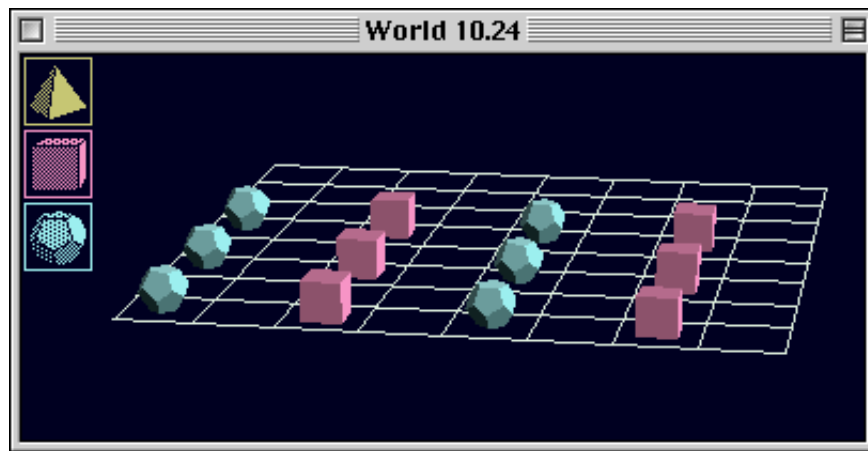
$$\begin{array}{|l} \text{Cube}(a) \\ \neg \text{Cube}(a) \\ \hline a \neq b \end{array}$$

Its truth functional form is:

$$\begin{array}{|l} A \\ \neg A \\ \hline B \end{array}$$

## Section 10.4 (page 265)

10.24 Here is a counterexample:



## Section 10.5 (page 270)

- 10.30
1.  $\neg \exists x (\text{Tet}(x) \wedge \text{Cube}(x))$
  2.  $\neg \exists x (\text{Cube}(x) \wedge \text{Dodec}(x))$
  3.  $\neg \exists x (\text{Dodec}(x) \wedge \text{Tet}(x))$
  4.  $\forall x (\text{Tet}(x) \vee \text{Dodec}(x) \vee \text{Cube}(x))$
  5.  $(\text{Cube}(a) \wedge \text{Dodec}(b)) \rightarrow a \neq b$  ✓ FO Con: 2
  6.  $\forall x (\text{Cube}(x) \rightarrow \neg \text{Tet}(x))$  ✓ FO Con: 1
  7.  $\forall x (\text{Small}(x) \rightarrow \text{Cube}(x)) \rightarrow \forall y (\text{Dodec}(y) \rightarrow \neg \text{Small}(y))$  ✓ FO Con: 2
  8.  $\forall x (\text{Large}(x) \rightarrow (\neg \text{Tet}(x) \wedge \neg \text{Dodec}(x) \wedge \neg \text{Cube}(x))) \rightarrow \neg \exists y \text{Large}(y)$  ✓ FO Con: 4