Chapter 5: Hints and Selected Solutions

Section 5.1 (page 131)

5.1 The pattern

$$\text{From } \mathsf{P} \vee \mathsf{Q} \text{ and } \neg\mathsf{P}, \text{ infer } \mathsf{Q}$$

is valid. If $\mathsf{P} \vee \mathsf{Q}$ is true, then by the truth table for $\vee$, at least one of P or Q must be true. But if $\neg\mathsf{P}$ is true, then by the truth table for$\neg$, P must be false. Hence, it must be that Q is true.

5.2 The pattern

$$\text{From } \mathsf{P} \vee \mathsf{Q} \text{ and } \mathsf{Q}, \text{ infer } \neg\mathsf{P}$$

is not valid. To see an example of how it could lead from true conclusions to false ones, consider the following argument:

> Washington or Lincoln was president of the U.S.
> Lincoln was president of the U.S.
> ---
> Washington was not president of the U.S.

This argument has true premises and false conclusion, but has the form of the displayed pattern. What makes the form tempting is the fact that people often read $\vee$ as exclusive disjunction.

Section 5.2 (page 134)

5.7 Here is an informal proof of the following argument:

> $\mathsf{Home}(\mathsf{max}) \vee \mathsf{Home}(\mathsf{claire})$
> $\neg\mathsf{Home}(\mathsf{max}) \vee \mathsf{Happy}(\mathsf{carl})$
> $\neg\mathsf{Home}(\mathsf{claire}) \vee \mathsf{Happy}(\mathsf{carl})$
> ---
> $\mathsf{Happy}(\mathsf{carl})$

Proof: Either Claire is not at home or Carl is happy, by the third premise. We break into cases. If Carl is happy, then we

have the desired conclusion. The other case is where Claire is not at home. By the first premise, either Claire or Max is at home, so, since we are assuming that Claire isn't, then Max is at home. But by the second premise, either Max is not at home or Carl is happy. So it must be that Carl is happy. So in either case, Carl is happy.

Section 5.3 (page 139)

5.16 We want to give a proof of the following argument:

> Max or Claire is at home but either Scruffy or Carl is unhappy.
> Either Max is not home or Carl is happy.
> Either Claire is not home or Scruffy is unhappy.
>
> ---
>
> Scruffy is unhappy.

Proof: We want to prove the conclusion using the method of proof by contradiction, so let us assume that Scruffy is not unhappy. By the third premise, then, it must be that Claire is not at home. But the first premise tells us, among other things, that either Claire or Max is at home. It would be a contradiction for Claire to be home, so Max must be. But by premise 2, Carl must therefore be happy. Again using premise 1, Scruffy must be unhappy. Thus we have our contradiction. This shows that our assumption that Scruffy was not unhappy must be false, so we have our desired conclusion, using proof by contradiction.

5.17 This argument is valid. A good way to start it is to use the third premise to break into two cases. The second of the two cases is quite easy. The first case takes a bit more thought.

5.23 We give a proof by contradiction. Assume that n^2 is odd but that n is even. But if n is even, it is of the form $2k$ for some integer k. But then $n^2 = 4k^2$, which is even since if we divide it by two, we get the integer $2k^2$. This contradicts our assumption that n^2 is odd.

5.26 Here is how the proof starts:

> Assume that $\sqrt{3}$ is rational. Then it can be expressed in the form p/q where p and q are whole numbers and at least one of them cannot be divided by 3. (Why?) Then, since $p/q = \sqrt{3}$, we can square each side and, with a bit of algebra, get $p^2 = 3q^2$. Now by the previous exercise, p^2 is divisible by 9.

Carry on!