

Chapter 4: Hints and Selected Solutions

Section 4.1 (page 104)

- 4.2 1. The truth table for $(A \wedge B) \vee (\neg A \vee \neg B)$ is shown in Figure ???. Since all the entries under the main connective (\vee) are T, it shows that the sentence is a tautology.

Table 4.2.1

=1=		=2=		(1)			
A	B	$(A \wedge B) \vee (\neg A \vee \neg B)$					
T	T	T	T	F	FF		
T	F	F	T	F	TT		
F	T	F	T	T	TF		
F	F	F	T	T	TT		

↑

- 4.5 The truth table is shown below. Since not all the entries under the main connective are T, the sentence is not a tautology. However, some of the entries in this column are T, so the sentence is TT-possible.

Table 4.5

\wedge \vee \neg \rightarrow \leftrightarrow \perp
 a b c d e f
 \forall \exists = \neq ()
 x y z u v w

Tet Small
 Cube Medium
 Dodec Large
 SameSize BackOf

Delete Column Verify Row
 Build Ref Cols Verify Table
 Fill Ref Cols Verify Assess

Correct? Complete? Assessment **Not Tautology, TT-Possible**

=1=	=2=	=3=	(1)
A	B	C	$A \vee \neg(B \vee \neg(C \wedge A))$
T	T	T	T F T F T
T	T	F	T F T T F
T	F	T	T T F F T
T	F	F	T F T T F
F	T	T	F F T T F
F	T	F	F F T T F
F	F	T	F F T T F
F	F	F	F F T T F

$\hat{1}$

4.9 Make sure you understand why there could not be a "Yes" in the first column and a "No" in the second. If you do not understand this, be sure to look it up in the book or ask your instructor. It is a simple but key point. Here are a few of the answers for this problem, so you can see if you have the right idea.

Sentence	TW-possible	TT-possible
1	Yes	Yes
4	No	Yes
7	No	Yes
10	Yes	Yes

Section 4.2 (page 109)

4.12 The truth table is shown below. Since the truth values under the main connectives of each sentence are the same, row by row, it shows that the sentences are tautologically equivalent.

Table 4.12

\wedge \vee \neg \rightarrow \leftrightarrow \perp
 a b c d e f
 \forall \exists = \neq ()
 x y z u v w

Tet Sma
 Cube Medit
 Dodec Larg
 SameSize Back

Delete Column Verify Row
 Build Ref Cols Verify Table
 Fill Ref Cols Verify Assess

Correct? Complete? Assessment **Tautologically Equivalent**

=1=	=2=	(1)	(2)
A	B	$\neg(A \vee B)$	$\neg A \wedge \neg B$
T	T	F	F
T	F	F	F
F	T	F	T
F	F	T	T

$\hat{1}$ $\hat{2}$

Section 4.3 (page 113)

4.21 The truth table is shown below. (We have abbreviated the sentences in order to make the picture fit on the page. You should not abbreviate yours, or GG will complain.) Since there is a row (the first) where the first two sentences are both true and the third is false, it shows that the third is not a tautological consequence of the first two.

Table 4.21

\wedge \vee \neg \rightarrow \leftrightarrow \perp
 a b c d e f
 \forall \exists = \neq ()
 x y z u v w

ed max
 e claire
 2:00
 2:05

Delete Column Verify Row
 Build Ref Cols Verify Table
 Fill Ref Cols Verify Assess

Correct? Complete? Assessment **Not \rightarrow Last**

=1=	=2=	(1)	(2)	(3)
$T(c, m)$	$T(m, c)$	$T(c, m) \vee T(m, c)$	$T(c, m)$	$\neg T(m, c)$
T	T	T	T	F
T	F	T	T	T
F	T	T	F	F
F	F	F	F	T

$\hat{1}$ $\hat{2}$ $\hat{3}$

Section 4.4 (page 116)

4.26 The complete proofs of Taut Con 1 and Taut Con 2 are shown below.

1. Home(max) \vee \neg Happy(carl)	
2. Happy(carl) \vee Hungry(carl)	
3. Hungry(carl) \vee Hungry(pris)	
4. Home(max) \vee Hungry(carl)	✓ ▾ Taut Con: 2,1
5. Hungry(carl) \vee (Home(max) \wedge Hungry(pris))	✓ ▾ Taut Con: 4,3
1. LeftOf(a, b) \vee (Dodec(a) \wedge Small(a))	
2. \neg LeftOf(a, b) \vee (Cube(b) \wedge \neg Large(b))	
3. \neg Small(a)	
4. Small(c)	
5. SameSize(a, b) \vee SameCol(a, b)	
6. $b = c \vee \neg$ (Cube(c) \vee \neg Dodec(c))	
7. LeftOf(a, b)	✓ ▾ Taut Con: 1,3
8. SameSize(a, b)	✓ ▾ Ana Con: 5,7
9. Cube(b) \wedge \neg Large(b)	✓ ▾ Taut Con: 7,2
10. \neg Large(b)	✓ ▾ Taut Con: 9
11. \neg Small(b)	✓ ▾ Ana Con: 3,8
12. Medium(b)	✓ ▾ Ana Con: 11,10
13. \neg (Cube(c) \vee \neg Dodec(c))	✓ ▾ FO Con: 11,4,6
14. \neg Cube(c) \wedge $\neg\neg$ Dodec(c)	✓ ▾ Taut Con: 13
15. Dodec(c)	✓ ▾ Taut Con: 14
16. Medium(b) \wedge Dodec(c)	✓ ▾ Taut Con: 15,12

Section 4.5 (page 120)

- 4.31
2. \neg Cube(a) \vee \neg Larger(b, a)
 4. \neg Cube(a) \wedge Larger(b, a)
 6. Cube(a) \wedge Larger(b, a) \wedge $a = b$
 8. \neg Tet(b) \wedge (\neg Large(c) \vee Smaller(d, e))
 10. Dodec(f) \vee (\neg Tet(b) \wedge Tet(f) \wedge Dodec(f))
(This sentence is logically equivalent to Dodec(f), so that would be a good answer, too.)

4.33

$$\begin{aligned}(A \wedge B) \wedge A &\Leftrightarrow A \wedge (B \wedge A) && \text{Associativity of } \wedge \\ &\Leftrightarrow A \wedge (A \wedge B) && \text{Commutativity of } \wedge \\ &\Leftrightarrow (A \wedge A) \wedge B && \text{Associativity of } \wedge \\ &\Leftrightarrow A \wedge B && \text{Idempotence of } \wedge\end{aligned}$$